INSTITUT UCAC-ICAM

Entrance Examination –August 29th 2020-International and Intercultural GENERALIST ENGINEER COURSE

To be filled by the candidate	Reserved for the Institute
Name : ……………………………..…… Surname : ……………….…………. Examination town : ………………… Seat N°: ……….. Subject : ………………………………………………………………	*Anonymous N°* : ………..

Reserved for the Institute	International and Intercultural GENERALIST ENGINEER COURSE	Reserved for the Institute
Score :	**MATHEMATICS**	*Anonymous N°* : :………..

INSTRUCTIONS: *There are 50 questions in this examination. Each question carries four (4) suggested answers, one of which is correct. Choose the correct answer among the alternatives.*

Calculators, booklets, etc are NOT allowed.

1. The derivative of the function $f(x) = e^{x^2} \sin \sin x$ is:
   A. $f'(x) = e^{x^2} \cos \cos x\,(1 + 2x \sin \sin x\,)$
   B. $f'(x) = e^{x^2} \sin \sin x\,(1 + 2x \sin \sin x\,)$
   C. $f'(x) = e^{x^2} \cos \cos x\,(1 + 2x \tan \tan x\,)$
   D. $f'(x) = e^{x^2} \sin \sin x\,(1 + 2x \tan \tan x\,)$

2. What is the tenth derivative, $f^{(10)}(x)$, of the function $f(x) = \frac{1}{x}$?
   A. $\frac{10!}{x^{10}}$
   B. $\frac{-10!}{x^{10}}$
   C. $\frac{-10!}{x^{11}}$
   D. $\frac{10!}{x^{11}}$

3. Which one of the following set is equal to $[\frac{1}{2}, 1) \cup [\frac{1}{3}, 1) \cup [\frac{1}{4}, 1) \cup \ldots$?
   A. $(0, 1)$
   B. $[0, 1)$
   C. $(0, 1]$
   D. $[0, 1]$

4. Under what condition is the following identity true, for $\theta_i \in R, i = 1, 2, 3$?
$$\sum_{i=1}^{3} \tan \tan \,_i = \prod_{i=1}^{3} \tan \tan \,_i$$
   A. $\sum \quad \theta_i = \frac{\pi}{4}$
   B. $\sum \quad \theta_i = \frac{\pi}{2}$
   C. $\sum \quad \theta_i = \pi$
   D. $\sum \quad \theta_i = 2\pi$

5. $$\sum_{n=1}^{1550} n(-1)^n =$$
   A. 225
   B. 775
   C. 1549
   D. 1550

6. What are the roots of the function $f(x) = \pi^3 - (\pi + \pi^2 + \pi^3)x + (1 + \pi + \pi^2)x^2 - x^3$?
   A. $\{\pi, \pi^2, \pi^3\}$
   B. $\{1, \pi, \pi^2\}$
   C. $\{-1, \pi, \pi^2\}$
   D. $\{-\pi, \pi^2, \pi^3\}$

7. The smallest value of the function $f(x) = x^2 + 2^{x^2}$, for $x \in R$ is:
   A. 0
   B. $\frac{1}{2}$
   C. 1
   D. 2

8. Which of the following is true about the inequality $tan\,tan\,x\ >sin\,sin\,x$, where $n \in N$?
   A. $\pi n < x < \pi n + \frac{\pi}{2}$
   B. $\pi n > x > \pi n + \frac{\pi}{2}$
   C. $\pi n > x > \left(\pi + \frac{\pi}{2}\right) n$
   D. $\pi n < x < \left(\pi + \frac{\pi}{2}\right) n$

9. The sum of the first n terms of the sequence whose terms are $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$ is:
   A. $6\left(2 - \left(\frac{2}{3}\right)^n\right)$
   B. $2 - \left(\frac{2}{3}\right)^n$
   C. $6\left(1 - \left(\frac{2}{3}\right)^n\right)$
   D. $6\left(1 - \left(\frac{3}{2}\right)^n\right)$

10. Given that

$$I = \int_0^{\pi} \frac{x\,sin\,sin\,x}{1 + x} dx$$

Then, one can show that:
   A. $f(x) = f(x - \pi)$
   B. $f(x) = f(x + \pi)$
   C. $f(x) = -f(x + \pi)$
   D. $f(x) = f(\pi - x)$

11. Upon evaluating the integral below, one obtains:

$$I = \int_0^{\pi} \frac{x\,sin\,sin\,x}{1 + x} dx$$

   A. $I = -\frac{\pi^2}{4}$
   B. $I = -\frac{\pi^2}{2}$
   C. $I = \frac{\pi^2}{2}$
   D. $I = \frac{\pi^2}{4}$

12. The term independent of x in the expansion of $\left(2x^2 + \left(\frac{1}{x^2}\right)\right)^8$ is:
   A. 1110
   B. 280
   C. 1120
   D. 340

13. The set of values of x that satisfies the equation $(x^3)\ - x^2(x^3)\ = x^2(x^3)$ is:
   A. $\{-1, 1\}$
   B. $\{-\sqrt{\pi}, \sqrt{\pi}\}$
   C. $\{-\pi, \pi\}$
   D. $\{-\pi^2, \pi^2\}$

14. The area and perimeter of a square of diagonal length 200m is:
   A. $40m^2$ and $3\sqrt{2}m$
   B. $20{,}000m^2$ and $400\sqrt{2}m$
   C. $4000\ m^2$ and $500m$
   D. $200\ m^2$ and $40\sqrt{2}m$

15. Simplifying
$cos\,cos\,(x\,) + (cos\,cos\,x\,) +$
$sin\,sin\,(x\,)\ +sin\,sin\,(x\,)$
yields:
   A. $\sqrt{x^2 - 1} + cos\,cos\,x$
   B. $\frac{x}{2} + 1 - 2x^2$
   C. $2\sqrt{x^2 + 1} - 4$
   D. $2(x + \sqrt{1 - x^2})$

16. The most correspondent identity for y, given that

$$y = \frac{cos\,cos\,6x\ + cos\,cos\,2x}{□□□\ sin\,6x\ + sin\,2x}, \qquad is:$$

   A. $tan\,tan\,2x\ + cot\,cot\,2x$
   B. $sec\,sec\,2x$
   C. $cot\,cot\,4x$
   D. $csc\,csc\,2x$

17. The derivative of

$f(x) = \left(\frac{a}{b}x\right)$, is:

A. $\frac{-a}{\sqrt{b^2-(ax)^2}}$
B. $\frac{-a}{\sqrt{b^2-ax^2}}$
C. $\frac{-a}{\sqrt{a^2-(bx)^2}}$
D. $\frac{-a}{\sqrt{a^2-b^2x}}$

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18. The expression $a^4 + b^4$ is best simplified as:
A. $\frac{1}{2}((a^2+b^2)^2 - (a^2-b^2)^2)$
B. $\frac{1}{2}((a^2-b^2)^2 - (a^2+b^2)^2)$
C. $\frac{1}{2}((a^2-b^2)^2 + (a^2-b^2)^2)$
D. $\frac{1}{2}((a^2+b^2)^2 + (a^2-b^2)^2)$

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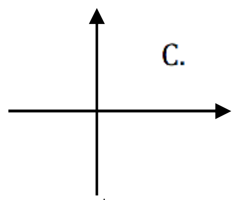
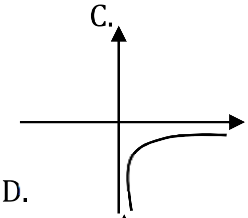
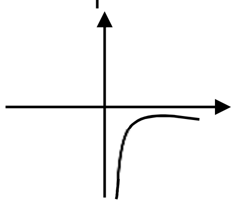
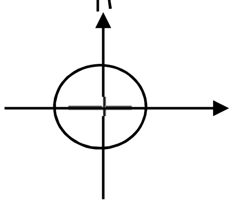
19. Which of the following curves represents a function?
A.


C.

B.

D.


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20. The coefficient of x^2 in the Maclaurin's series expansion of $ln\, ln\, (1-2x)$ is:
A. $\frac{1}{2}$
B. 1
C. -2
D. 2

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21. If $cos\, cos\, (2x) = 1$, then $cos\, cos\, x =$:
A. 0
B. 1
C. -1
D. $\frac{1}{2}$

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22. Given that $r = pe^{-2t}$ is a particular solution of the vector differential equation

$$\frac{d^2r}{dt^2} + \frac{dr}{dt} = (4i - 12j)e^{-2t},$$

Then the value of p is:
A. $-4i + 12j$
B. $-2i + 6j$
C. $\frac{3}{2}i - 2j$
D. $2i - 6j$

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23. Given that $\lambda, \mu \in R$, then the value of

$$\left(\frac{x + \lambda x^2}{x^2 + \mu x}\right) =:$$

A. ∞
B. 1
C. λ
D. λ/μ

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24. At what point is the function f discontinous, where f is given by:

$$f(x) = \{1 - 2x^2,\ \ 0 \le x < 1\, 3 - 4x,\ \ 1 \le x < 2\, x^2 - 9,\ \ 2 \le x \le 3\, 0,\ \ \ \ elsewhere\ ?$$

A. $x = 0$
B. $x = 1$
C. $x = 2$
D. $x = 3$

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25. The oblique asymptote to the curve

$$y = \frac{3x^2}{2-x}, \quad is:$$

A. $y = 3x + 6$
B. $y = -3x + 6$
C. $y = -3x - 6$
D. $y = 3x - 6$

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26. The root mean square of $\sqrt{x}$ over the interval $0 \leq x \leq 2$ is:
   A. $\frac{2}{3}\sqrt{2}$
   B. $\sqrt{2}$
   C. 1
   D. $\frac{1}{3}\sqrt{2}$

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27. Given that
$$I_n = \int_0^1 x^n e^x dx, \qquad then\ I_n =$$
   A. $e - I_{n-1}$
   B. $e - nI_{n-1}$
   C. $e + I_{n-1}$
   D. $e + nI_{n-1}$

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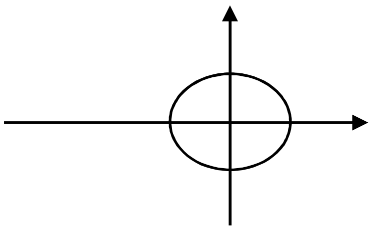
28. The shaded region on the Cartesian plane represents the regions described by the inequality:
   A. $(x^2 + y^2 - 1)(y^2 + 4x) \geq 0$
   B. $(x^2 + y^2 - 1)(y^2 + 4x) \leq 0$
   C. $(x^2 + y^2 + 1)(y^2 + 4x) \geq 0$
   D. $(x^2 + y^2 + 1)(y^2 - 4x) \geq 0$



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29. The series
$$\sum_{r=1}^{\infty} \left(\frac{1}{r}\right)^n$$
converges for:
   A. $n > 1$
   B. $n \geq 1$
   C. $n < 1$
   D. $n \leq 1$

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30. If f is an odd function, then,
$$\int_{-a}^{a} f(x)dx =$$
   A. 0
   B. 1
   C. $\int_0^a f(x)dx$
   D. $2\int_0^a f(x)dx$

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31. The Maclaurin expansion for $ln\, ln\, (3 + 4x)$ is valid for:
   A. $-\frac{3}{4} \leq x \leq \frac{3}{4}$
   B. $-\frac{3}{4} < x \leq \frac{3}{4}$
   C. $-\frac{3}{4} \leq x < \frac{3}{4}$
   D. $-\frac{3}{4} < x < \frac{3}{4}$

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32. The moment of inertia, I_G, of a solid sphere, of mass m and radius a, about its center is $\frac{2}{5}ma^2$. The moment of inertia of the sphere about an axis through its tangent is:
   A. $\frac{4}{3}ma^2$
   B. ma^2
   C. $\frac{3}{5}ma^2$
   D. $\frac{7}{5}ma^2$

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33. Evaluating
$$\left(x\, sin\, sin\, \left(\frac{1}{x}\right)\right), \qquad gives:$$
   A. -1
   B. 0
   C. 1
   D. ∞

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34. A particle, P moves round a curve $r = a\theta$, such that OP rotates with constant angular velocity ω. The transverse component of the velocity when P is at the point (r, θ) is:

A. $\frac{r}{\omega}$
B. $r\omega$
C. $r^2\omega$
D. $r\omega^2$

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35. Given that $f(x) = ln\ ln\ (sinh\ sinh\ 2x\ )$, then, the derivative, $f'(x)$, of $f(x)$ is:
A. $2\ csc\ csc\ 2x$
B. $2\ cosh\ cosh\ 2x$
C. $2\ coth\ coth\ 2x$
D. $ln\ ln\ (2\ cosh\ cosh\ 2x\ )$

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36. The value of k that renders the function $f(x)$ continuous, where
$f(x) = \{kx^2 + 4, \quad x < 1\ (x+3)^2, \quad x \geq 1\ , \quad is:$
A. 16
B. -12
C. -16
D. 12

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37. If $f: [a, b] \to R$ is strictly monotonic on $[a, b]$ and the function f has a root $x_0 \in (a, b)$, then:
A. $f(a) \cdot f(b) = 0$
B. $f(a) \cdot f(b) > 0$
C. $f(a) \cdot f(b) < 0$
D. $f(a) = f(b)$

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38. Let f, g and h be functions such that
$f(x) = \{g(x), \quad x < a\ h(x), \quad x \geq a$
Then f is continuous at $x = a$ if:
A. $h(x) = g(a) = f(a)$
B. $f(a) = g(a) = h(a)$
C. $h(x) = \lim_{x \to a^-} g(x) = f(a)$
D. $\lim_{x \to a^-} h(x) = \lim_{x \to a^+} g(x) = f(a)$

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39. The equation of curve is parametrically given by $x = r\ cos\ cos\ ⍰$, $y = r\ sin\ sin\ ⍰$, where θ is a parameter, and r a constant. The length of the curve, from $\theta = 0$ to $\theta = 2\pi$ is:
A. πr
B. $\frac{3}{4}\pi r$
C. $2\pi r$
D. $4\pi r$

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40. A particle moves along a straight line OX, such that its displacement x from the equilibrium point O satisfies the differential equation $\ddot{x} + 4\dot{x} + 5x = 0$. The period of the motion is:
A. π
B. $\frac{\pi}{2}$
C. 2π
D. $\frac{\pi}{5}$

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41. Evaluating $\int \quad x\ dx$ yields:
A. $\frac{1}{4}(2x + 2x) + c$
B. $\frac{1}{2}(sinh\ sinh\ 2x + 2x) + c$
C. $\frac{1}{4}(sinh\ sinh\ 2x - 2x) + c$
D. $\frac{1}{2}(sinh\ sinh\ 2x - 2x) + c$

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42. Given the differential equation $y'' + 3y' - 4y = 0$, the general solution of the differential equation, where $A, B \in R$, is:
A. $A\ cos\ cos\ 4x + B\ sin\ sin\ x$
B. $Ae^x\ cos\ cos\ (4x + \epsilon)$
C. $Ae^x + Be^{-4x}$
D. $(A + Bx)e^{-4x}$

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43. Which of the following curves does not have a vertical asymptote?
A. $f(x) = \frac{x^2+3x}{x+3}$
B. $f(x) = \frac{x^2+x}{x+3}$

C. $f(x) = \frac{x^2-x-6}{x+3}$

D. $f(x) = ln\ ln\ x$

44. The modulus of the complex number $z = 1 - e^{i\theta}$ is:

A. $2\ sin\ sin\ \left(\frac{\square}{2}\right)$

B. $2\ cos\ cos\ \left(\frac{\square}{2}\right)$

C. 2

D. 1

45. Evaluating

$$\left(1 - \frac{1}{n}\right)^{-n} =$$

A. ∞

B. e

C. 0

D. $-e$

46. Which of the following series converges?

A. $\sum_{r=0}^{\infty} \quad (1-r)^r$

B. $\sum_{r=0}^{\infty} \quad r(1+r)^r$

C. $\sum_{r=0}^{\infty} \quad \left(\frac{1}{r}\right)$

D. $\sum_{r=0}^{\infty} \quad \left(\frac{3^r}{r!}\right)$

47. Consider the function

$$f(x) = \int_a^x \frac{t^2}{\sqrt{t^2+1}} dt$$

Then, for $a \geq 0$, the function f:

A. Is monotone increasing

B. Has a turning point

C. Is monotone decreasing

D. Is concave downwards

48. The centre is symmetry of the curve is

$$f(x) = \frac{1}{x-2}$$

at the point with Cartesian coordinates:

A. $(0, 0)$

B. $(-2, 0)$

C. $(2, 0)$

D. $(0, 2)$

49. If $A, B \in R$, a particular solution of the differential equation $y''(x) + 9y(x) = sin\ sin\ 3x$, is of the form:

A. $y = A\ sin\ sin\ x\ + B\ cos\ cos\ x$

B. $y = A\ sin\ sin\ 3\ + B\ cos\ cos\ 3x$

C. $y = Ax\ cos\ cos\ 3x$

D. $y = Ax\ sin\ sin\ 3x$

50. The moment of inertial, I of a particle of mass $2m$ is $\frac{8}{3}ma^2$. Its radius of gyration, k, is:

A. $\frac{2a}{3}\sqrt{3}$

B. $\frac{a}{3}\sqrt{3}$

C. $\frac{4}{3}a^2$

D. $\frac{1}{2a}\sqrt{3}$