INSTITUT UCAC-ICAM

Entrance Examination –June 2020-International and Intercultural GENERALIST ENGINEER COURSE

To be filled by the candidate	Reserved for the Institute
Name : ……………………………..…… Surname : ……………….………… Examination town : ………………… Seat N°: ………. Subject : ……………………………………………………………………	*Anonymous N°* : ……….

Reserved for the Institute	International and Intercultural GENERALIST ENGINEER COURSE	Reserved for the Institute
Score :	**MATHEMATICS**	*Anonymous N°* : : ……….

1. Given that $A, B, C, D \in \mathbb{R}$, the partial fraction decomposition of $f(x) = \frac{x^2}{(x^2+1)^2}$ is:
   A. $\frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+2)^2}$
   B. $\frac{Ax+B}{x^2+1} + \frac{Cx^2+D}{(x^2+1)^2}$
   C. $\frac{Ax}{x^2+1} + \frac{B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$
   D. $\frac{A}{x^2+1} + \frac{Bx}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$

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2. Evaluating $\int \sinh^2 x \, dx$ yields:
   A. $\frac{1}{4}(\sinh\ 2x + x) + c$
   B. $\frac{1}{2}(\sinh 2x + x) + c$
   C. $\frac{1}{4}(\sinh 2x - x) + c$
   D. $\frac{1}{2}(\sinh 2x - x) + c$

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3. Given the differential equation $y'' + 3y' - 4y = 0$, the general solution of the differential equation, where $A, B \in \mathbb{R}$, is:
   A. $A \cos 4x + B \sin x$
   B. $Ae^x \cos(4x + \epsilon)$
   C. $Ae^x + Be^{-4x}$
   D. $(A + Bx)e^{-4x}$

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4. Which of the following curves does not have a vertical asymptote?
   A. $f(x) = \frac{x^2+3x}{x+3}$
   B. $f(x) = \frac{x^2+x}{x+3}$
   C. $f(x) = \frac{x^2-x-6}{x+3}$
   D. $f(x) = \ln x$

5. The modulus of the complex number $z = 1 - e^{i\theta}$ is:
   A. $2\sin\left(\frac{\theta}{2}\right)$
   B. $2\cos\left(\frac{\theta}{2}\right)$
   C. 2
   D. 1

6. Which of the following series converges?
   A. $\sum_{r=0}^{\infty}(1-r)^r$
   B. $\sum_{r=0}^{\infty} r(1+r)^r$
   C. $\sum_{r=0}^{\infty}\left(\frac{1}{r}\right)$
   D. $\sum_{r=0}^{\infty}\left(\frac{3^r}{r!}\right)$

7. Consider the function

$$f(x) = \int_a^x \frac{t^2}{\sqrt{t^2+1}}\,dt$$

   Then, for $a \geq 0$, the function f:
   A. Is monotone increasing
   B. Has a turning point
   C. Is monotone decreasing
   D. Is concave downwards

8. The area under the curve with parametric equation $x = 1 + t^2$, $y = t(2-t)$, for $t \in [0,1]$ is:
   A. $\frac{5}{3}$
   B. $\frac{5}{6}$
   C. $\frac{6}{5}$
   D. $\frac{2}{3}$

9. The equation $25x^2 + y^2 - 100x - 2y + 76 = 0$ represeents:
   A. A parabola
   B. A circle

C. An ellipse
D. A hyperbola

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10. Given that $y = \lambda x \sin x$ is a particular integral of the differential equation $y'' + y = \cos x$, then the value of λ is:
A. π
B. 1
C. $\frac{1}{2}$
D. 2

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11. Given that the equation of motion along OX after time t is $x''(t) + 4x(t) = 0$, then the period of the motion is:
A. π
B. $\frac{\pi}{2}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{8}$

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12. If a force $\boldsymbol{F}$ acts through a point with position vector $\boldsymbol{a}$, then its moment about a point with position vector $\boldsymbol{b}$ is given by:
A. $\boldsymbol{a} \times \boldsymbol{F}$
B. $(\boldsymbol{b} - \boldsymbol{a}) \times \boldsymbol{F}$
C. $(\boldsymbol{a} - \boldsymbol{b}) \times \boldsymbol{F}$
D. $\overline{0}$

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13. Which of the following statements is true about the function f defined by:

$$f: x \mapsto \begin{cases} \dfrac{x^2 - 9}{x - 3}, x \neq 3 \\ 3, \qquad x = 3 \end{cases} \ ?$$

A. f is continuous at $x = 3$
B. f is not defined at $x = 3$
C. $\lim_{x \to 3} f(x) = 3$
D. f is defined at $x = 3$

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14. If $y'(x) - 2xy(x) - 1 = 0$ and that when $y = 0, x = 0$, the first two non-zero terms in the series for $y(x)$ are:
    A. $1 + x$
    B. $x - \frac{x^2}{2}$
    C. $x - x^2$
    D. $x + \frac{2x^2}{3}$

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15. A particle moves round a polar curve $r = a + a\cos\theta$ with constant angular velocity ω. The transverse component of the velocity is:
    A. $a\omega$
    B. ωr
    C. $-a\omega\sin\theta$
    D. $a\omega - \sin\theta$

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16. A sphere traveling on a smooth horizontal floor with speed $u\ m/s$ hits a vertical wall at an angle of 45^0. Given that the coefficient of restitution between the sphere and the wall is e, the speed of the sphere after impact is:
    A. $e\sqrt{34}$
    B. $2e\sqrt{2}$
    C. $3e\sqrt{2}$
    D. $e\sqrt{3}$

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17. The series

$$\sum_{n=1}^{\infty}\left(\frac{5}{2n}\right)^r$$

converges when:
    A. $r > 1$
    B. $r < 1$
    C. $r = 1$
    D. $r = 0$

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18. The centre is symmetry of the curve is

$$f(x) = \frac{1}{x-2}$$

at the point with Cartesian coordinates:

A. $(0,0)$
B. $(-2,0)$
C. $(2,0)$
D. $(0,2)$

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19. If $A, B \in \mathbb{R}$, a particular solution of the differential equation $y''(x) + 9y(x) = \sin 3x$, is of the form:
    A. $y = A\sin x + B\cos x$
    B. $y = A\sin 3 + B\cos 3x$
    C. $y = Ax\cos 3x$
    D. $y = Ax\sin 3x$

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20. The moment of inertial, I of a particle of mass $2m$ is $\frac{8}{3}ma^2$. Its radius of gyration, k, is:
    A. $\frac{2a}{3}\sqrt{3}$
    B. $\frac{a}{3}\sqrt{3}$
    C. $\frac{4}{3}a^2$
    D. $\frac{1}{2a}\sqrt{3}$

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21. The Cartesian equation of the polar curve $r^2 = 2a^2\sin 2\theta$ is:
    A. $x^2 + y^2 = 4a^2xy$
    B. $x^2 + y^2 = a^2xy$
    C. $x^2 + y^2 = 4xy$
    D. $(x^2 + y^2)^2 = 4a^2xy$

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22. The range of the function $y = \frac{x}{x^2+1}$ is:
    A. $-\frac{1}{2} \leq y \leq \frac{1}{2}$
    B. $-\frac{1}{2} \leq y < \frac{1}{2}$
    C. $y \leq -\frac{1}{2}, y \geq \frac{1}{2}$
    D. $y < -\frac{1}{2}, y > \frac{1}{2}$

23. Given that $k \in \mathbb{R}$ nd that

$$\lim_{x \to x_0} \frac{\sqrt{1+f(x)}}{x} = l, \qquad \text{then} \quad \lim_{x \to x_0} \frac{\sqrt{1+f(kx)}}{x} = ?$$

A. $\sqrt{kl}$
B. kl
C. $\frac{l}{k}$
D. $\frac{k}{l}$

24. The parametric equation o the rectangular hyperbola $xy + x = c^2$ is:

A. $x = ct; y = \frac{c}{t} - 1$
B. $x = 1 + ct; y = \frac{c}{t}$
C. $x = ct; y = \frac{c}{t}$
D. $x = ct; y = \frac{c}{t} + 1$

25. The mean value of

$$f(x) = \frac{1}{1+4x^2}, \qquad \text{for } 0 \le x \le \frac{1}{2} \quad \text{is:}$$

A. $\frac{\pi}{16}$
B. $\frac{\pi}{8}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{2}$

26. Given that $\alpha = \arctan\left(\frac{1}{2}\right)$ and $\beta = \arctan\left(\frac{1}{3}\right)$, then the value of $\alpha - \beta$ is:

A. $\arctan\left(\frac{1}{7}\right)$
B. $\arctan\left(\frac{1}{5}\right)$
C. $\arctan\left(\frac{5}{7}\right)$
D. $\arctan(1)$

27.

$$\sum_{k=1}^{2n}(-1)^k =$$

A. ∞
B. 1
C. -1
D. 0

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28. The equation of the asymptote to the curve $y^3 = 6x^2 - x^3$ is:
A. $y = x$
B. $y = -x$
C. $y = 2 - x$
D. $y = x + 2$

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29. Evaluating

$$\int_0^{\ln 2} e^{\cosh x} \sinh x \, dx \quad \text{yields}$$

A. $e^{\frac{1}{4}} - e$
B. $e^{\frac{5}{2}} - e$
C. $e^{\frac{5}{4}} - e$
D. $e^{\frac{1}{4}} - e$

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30. The greatest common divisor of the function $f(x) = x^2 + 3x$ and $g(x) = 3x^3 + 6x^2$, for $x > 0$ is:
A. $x^2 + 2x$
B. $3x + 6$
C. $x + 2$
D. x

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31. The oblique asymptote to the curve

$$y = x - 2 + \frac{1 - x}{5 + x}, \quad \text{is:}$$

A. $y = x - 2$
B. $y = x - 1$
C. $y = x - 3$

D. $y = x$

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32. Given the sequence $\{u_n\}$, defined by $u_n := n^2\left(1 - \cos\left(\frac{1}{n}\right)\right)$, then
$$\lim_{n\to\infty} u_n =$$
A. 1
B. $\frac{1}{2}$
C. 0
D. $+\infty$

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33. The polar equation $r = -8\cos\theta$ represents:
A. A parabola
B. A hyperbola
C. An ellipse
D. A circle

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34. Evaluating
$$\int \frac{1}{9x^2+4}\,dx, \qquad \text{yields}$$
A. $\frac{1}{6}\arctan\left(\frac{3}{4}x\right) + c$
B. $\frac{1}{3}\arctan\left(\frac{3}{4}x\right) + c$
C. $\frac{1}{6}\arctan\left(\frac{3}{2}x\right) + c$
D. $\frac{1}{3}\arctan\left(\frac{3}{2}x\right) + c$

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35. Given that the vector $\boldsymbol{i} + \lambda\boldsymbol{j} + \mu\boldsymbol{k}$ is parallel to the planes $\pi_1: 2x + 3y + 4z = 3$ and $\pi_2: x + 2y + 3z = 2$, then the values of λ and μ are respectively:
A. 1 and -2
B. -1 and 2
C. -2 and 11
D. -2 and 1

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36. The equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8x = 0$$

represents an oscillatory motion. The period, T, of this motion is:

A. 4π
B. 3π
C. 2π
D. π

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37. Evaluating $\arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{3}\right)$ yields:

A. $\arctan\left(\frac{5}{6}\right)$
B. $\arctan\left(\frac{1}{6}\right)$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{2}$

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38. The argument of the complex number $z = \frac{1+i}{\sqrt{3}} + i$ is:

A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{5\pi}{12}$
D. $\frac{\pi}{12}$

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39. If $\sinh x = 2$, then $x =$

A. $\ln(2 + \sqrt{3})$
B. $\ln(2 - \sqrt{3})$
C. $\ln(2 + \sqrt{5})$
D. $\ln(2 - \sqrt{5})$

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40. How many divisors does the number 2020 have?

A. 12
B. 20
C. 64
D. 256

41. If $g(x) = [x]$ denotes the greatest integer function, and that $f(x) = 1 + g(x)$, then
$$\int_0^3 f(x)\,dx =$$
   A. 3
   B. 7
   C. 2
   D. 4

42. A particle of mass m, moves so that at time t, its position vector is given by $r = \cosh 2t\,\boldsymbol{i} + \sin 2t\,\boldsymbol{j}$. The moment of momentum of the particle about the origin is:
   A. $-2m\boldsymbol{j}$
   B. $-2m\boldsymbol{i}$
   C. $-2m(\boldsymbol{i} + \boldsymbol{j})$
   D. $-2m$

43. Evaluating
$$\frac{(\cos 2x + i \sin 2x)^3}{(\cos 2x - i \sin 2x)^{-2}}, \qquad \text{yields:}$$
   A. $\cos 2x + is \sin 2x$
   B. $\cos 5x + i \sin 5x$
   C. $\cos 8x + i \sin 8x$
   D. $\cos 10x + i \sin 10x$

44. The Cartesian equation of the polar curve $r^2 \sin 2\theta = 1$ is:
   A. $xy = 1$
   B. $2xy = 1$
   C. $y(x^2 + y^2) = 1$
   D. $xy = 2$

45. The energy equation of a compound pendulum is $2a\,\dot{\theta}^2 = 3g \cos\theta$. The period of small oscillations of the pendulum is:
   A. $4\pi\sqrt{\frac{a}{3g}}$

B. $2\pi\sqrt{\frac{2a}{3g}}$

C. $\sqrt{\frac{3a}{g}}$

D. $\sqrt{\frac{4a}{3g}}$

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46. Evaluating sinh(ln 2) yields:
   A. 0
   B. $\frac{3}{4}$
   C. $\frac{5}{4}$
   D. $\frac{3}{2}$

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47. The gradient of the normal to the rectangular hyperbola $xy = p^2$ at the point $\left(pt, \frac{p}{t}\right)$ is:
   A. $\frac{1}{t^2}$
   B. t^2
   C. $-\frac{1}{t^2}$
   D. $-t^2$

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48. The period of the compound pendulum is $T = 4\pi\sqrt{\frac{a}{3g}}$. The length of the equivalent simple pendulum is:
   A. $\frac{4a}{3}$
   B. $\frac{2a}{3}$
   C. $\frac{4a}{3g}$
   D. $\frac{2a}{3g}$

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49. The differential equation $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + n^2x = 0$ represents damped harmonic motion if:
   A. $k^2 = n^2$
   B. $k^2 > n^2$
   C. $2k = n^2$
   D. $k^2 < n^2$

50. The number x, of bacteria in a culture reduces at a rate equal to twice the number of bacteria present at time t. The differential equation describing this situation, where k, is a constant of proportionality, is:

    A. $\frac{dx}{dt} - 2kx = 0$

    B. $\frac{dx}{dt} + 2kx = 0$

    C. $\frac{dx}{dt} - \frac{1}{2}kx = 0$

    D. $\frac{dx}{dt} + \frac{1}{2}kx = 0$