UCAC-ICAM INSTITUTE

Entrance Examination- May 2018 – International and Intercultural GENERALIST ENGINEER COURSE

To be filled by the Candidate :	Reserved for the Institute
Name : ……..… Surname : …….……	*Anonymous N°* :
Examination town : …… Seat N° : …….	……..
Subject : Mathematics	

Reserved for the Institute	☑ International and Intercultural GENERALIST ENGINEER COURSE.	Reserved for the Institute
Score :	**Mathematics Paper – Time : 1 h 30** Calculators and documents are not allowed– Number of pages : 9 Answer only on the answering sheet; to be submitted at the end-time equally with the question sheet.	*Anonymous N° :* ……..

Start by filling in your names, surnames examination center and your seat number below.

Answer directly on the answering sheet which must be submitted at the end of the exam as well as the question booklet. Any working should be done in this question booklet.

Invigilators have the right to exclude from the competition any candidate trying to copy from another candidate, or who is caught with any document, or who begins writing before the start-up time or who continues to write after the end of the time.

Notice :
Carefully read the text and answer all the questions below.
For each question there are four suggested answers A, B, C and D. Choose the one you consider correct.
For every right answer, you will be allocated (0.4 mark) ;
For any wrong answer, you will be subtracted (0.1 mark). No answer = 0 mark

Define a real valued function f by:

$$f(x) = \frac{\sqrt{x+3}-\sqrt{3}}{x}.$$

Answer questions 1 and 2.

1. The domain, D_f of f is:
   a. $D_f := \{x \neq 0, x \geq 3\}$
   b. $D_f := \{x \neq 0, x \geq -3\}$
   c. $D_f := x: x \in [-3,0[\cup]0,\infty[$
   d. $D_f := \{x: x \notin \mathbb{R}\{-3,0\}\}$

2. $\lim_{x\to 0} f(x)$ is:
   a. $\frac{1}{2\sqrt{3}}$ c. $\sqrt{3}$
   b. 0 d. $\nexists$

3. The derivative of
   $f(x) = (1 + x^4 - ln x)^{5/3}$ is:
   a. $\frac{20}{3}x^{\frac{17}{3}} + \frac{5}{3}(ln x)^{8/3}$
   b. $\frac{5}{3}(1 + x^4 - ln x)^{\frac{2}{3}}(4x^3 - \frac{1}{x})$
   c. $\frac{5}{3}(1 + x^4 - ln x)^{2/3}$
   d. $\frac{5}{3}x^{\frac{2}{3}}(4x^3 + \frac{1}{x^2})$

4. For what value of k is the function $f(x) = \begin{cases} kx^2 + 2x, & if\ x < 2 \\ 2x + 4, & if\ x \geq 2 \end{cases}$ continues on $\mathbb{R}$?
   a. 1 b. 0 c. 2 d. 4

5. An equation of the tangent line to the curve $y = 2sinx$ at the point:
   a. $y = 2x + 2\pi$
   b. $y = 2x$
   c. $y = 2(\pi - x)$
   d. $y = -2x$

6. The Maclaurin series expansion of a function f is given by $f(x) =$:
   a. $f(x) + xf'(x) + \frac{x^2}{2!}f''(x) + \cdots$
   b. $f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$
   c. $f(x) - xf'(x) + \frac{x^2}{2!}f''(x) - \cdots$
   d. $1 + x + \frac{x^2}{2!} + \cdots$

7. The series expansion of $tanx$ in ascending powers of x is given by:
   a. $1 + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$
   b. $1 + x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$
   c. $x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \cdots$
   d. $x + \frac{x^3}{3} + \frac{x^3}{5} - \frac{x^6}{6} + \cdots$

8. The series expansion of e^{-x} is:
   a. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
   b. $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$
   c. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
   d. $x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

9. $\lim_{x\to\pi} \frac{1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\cdots}{1/x}$ yields:
   a. ∞ b. 0 c. 1 d. -1

10. $\lim_{x\to\pi} \frac{\sqrt{x}-\sqrt{\pi}}{x-\pi}$ gives
   a. ∞ b. 0 c. $\frac{1}{2\sqrt{x}}$ d. π

11. The Taylor series expansion of $f(x+a)$ is:
   a. $f(a) + af'(a) + \frac{a^2}{2!}f''(a) + \cdots$
   b. $f(a) - xf"(a) + \frac{a}{2!}f'(a) - \cdots$
   c. $f(x) + af'(x) + \frac{a^2}{2!}f''(x) + \cdots$
   d. $f(0) + af"(0) + \frac{a^2}{2!}f'(0) + \cdots$

12. After covering a distance of 25π units, a car's wheel completed 25 revolutions. What is the diameter of the car's wheel in units is?
   a. π b. 1 c. 2 d. 0.25

13. The graph of the function $y = sinx$ is shifted a distance $\frac{\pi}{2}$ to the left, reflected in the x-axis, translated 5 units downward, then stretched by a factor of 4. The resulting function is:
   a. $y = 4\sin\left(x + \frac{\pi}{12}\right) - 20$
   b. $y = -4\sin\left(x + \frac{\pi}{12}\right) - 20$
   c. $y = 4\sin\left(x + \frac{\pi}{12}\right) - 5$
   d. $y = -4\sin\left(x + \frac{\pi}{12}\right) - 5$

14. Which of the following series converges?

a. $\sum_{r=0}^{\infty} \frac{2^r}{r!}$
b. $\sum_{r=0}^{\infty} \frac{1}{r}$
c. $\sum_{r=0}^{\infty} (-1)^r$
d. $\sum_{r=0}^{\infty} (1-r)^r$

15. The phase shift of the function
$y = \cos(x - \frac{\pi}{2})$ is:
a. $\frac{\pi}{2}$ b. $-\frac{\pi}{2}$ c. 2π d. π

16. Simplifying $sec x(cot x + sin x)$ yields:
a. $\sin x + \csc x$
b. $\csc x + \tan x$
c. $\cos^2 x + 2\sin^2 x$
d. $\cot^2 x - \cos x$

17. Expressing $\cos\theta$ in terms of $\csc\theta$ yields:
a. $\pm\sqrt{\csc^2\theta - 1}$
b. $\frac{1}{\csc\theta}$
c. $\pm\frac{\sqrt{\csc^2\theta - 1}}{csc\theta}$
d. $\pm\frac{\csc\theta\sqrt{\csc^2\theta - 1}}{\csc^2\theta + 1}$

18. $\int_0^3 |x(1-x)|dx$ yields:
a. $1/6$ c. $9/2$
b. $29/6$ d. $55/6$

19. Given that $2y^2 = \tan x \sin 2x$, then expressing y in terms of x yields:
a. $|\sin x|$
b. $\tan^{-1}(\sin^{-1}(y^2))$
c. $\sin^{-1}(\tan^{-1}(y^2))$
d. $\sin x$

20. Define a function g on $\mathbb{C}$ by:
$g(x) = x^2 - 8x + 20$. $g(4 + 2i)$ is:
a. $4i$ b. i c. 4 d. 0

21. Define the functions:
$f(x) = e^x$, $g(x) = \ln x$,
$h(x) = \sin x$, $i(x) = x$. Then
$\lim_{x\to 0} \frac{f\circ g\circ h\circ i}{i}$ is:
a. 1 b. 0 c. ∞ d. -1

22. Which of the following series converges?
a. $\sum \frac{x^n}{n!}$ c. $\sum \frac{1}{n+\sin n}$
b. $\sum (-1)^n n$ d. $\sum \sin n$

23. Given that $\cos\theta = -\frac{3}{2}$ and $\tan\theta > 0$, then the value of $\cos 2\theta$ is:
a. $-\frac{1}{9}$ c. $-\frac{4\sqrt{5}}{9}$
b. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

24. Given that f is an even function and that $I = \int_{-a}^{a} f(x)dx$, $a > 0$, which of the following is true?

a. $\frac{1}{2}I = \int_0^a f(x)dx$
b. $I = 0$
c. $I = \int_0^a f(x)dx$
d. $I = 2f'(a)$

25. The two asymptotes of the curve
$f(x) = \frac{x^2}{1-3x}$ are:
a. $x = \frac{1}{3}, f(x) = \frac{1}{9} - x$
b. $x = \frac{1}{3}, f(x) = \frac{x}{3} - \frac{1}{9}$
c. $x = -\frac{1}{3}, f(x) = -\frac{1}{3} - \frac{1}{9}$
d. $x = -\frac{1}{3}, f(x) = \frac{x}{3} + \frac{1}{9}$

26. Using Simpson's Rule with three ordinates, $\int_1^{49} \frac{1}{\sqrt{x}} dx$ yields:
a. $\frac{216}{35}$ c. $\frac{272}{35}$
b. $\frac{144}{35}$ d. $\frac{554}{35}$

27. Given that $f(x) = |x-1| + |x-4|$, evaluating then $f'(5)$ yields:

a. -5 c. -3

b. 3 d. 2

28. $\lim_{x\to 0}\left(\frac{x^2+2\cos x-2}{x^4}\right)$ yields:

a. $\frac{1}{2}$ c. $\frac{1}{4}$

b. $-\frac{1}{2}$ d. $\frac{1}{12}$

29. The expansion of e^{x+x^2} as a series in ascending powers of x as far as the term in x^3 yields:

a. $1 + x + x^2 + x^3$

b. $1 + x + \frac{x^2}{2} + \frac{x^3}{3}$

c. $1 + x + \frac{3x^2}{2} + \frac{4x^3}{3}$

d. $1 + x + \frac{3x^2}{2} + \frac{7x^3}{6}$

30. The integrating factor of the differential equation $(x-1)f'(x) + xy = (x-1)e^x$ is:

a. $x - 1$ c. $\frac{x^2}{2}$

b. $(x-1)e^x$ d. $(x-1)e^{-x}$

31. The equation $\cos x + \sqrt{3}\sin x = 1$ is equivalent to:

a. $2\sin\left(x + \frac{\pi}{6}\right) = 1$

b. $2\sin\left(x + \frac{\pi}{3}\right) = 1$

c. $2\cos\left(x - \frac{\pi}{6}\right) = 1$

d. $2\cos\left(x - \frac{\pi}{3}\right) = 1$

32. Given that $\sinh x = \frac{4}{3}$, then $\cosh\frac{x}{2} =$

a. $\frac{2\sqrt{3}}{3}$

b. $\ln 3$

c. $\frac{5}{3}$

d. $\frac{\sqrt{3}}{3}$

33. Given the complex number z, where $z = a + bi, a, b \in \mathbb{R}^*$,
which of the following is not true about z.
a. $|zz^*| = |z|^2$
b. $(z^n)^* = (z^*)^n$
c. $|z + z^*| = |z| + |z^*|$
d. $\arg(z^*) = -\arg(z)$

34. If the cube roots of unity are $1, \omega \text{ and } \omega^2$, then the value of $\left(\frac{1+\omega}{\omega^2}\right)^3$ is:
a. 1 b. -1 c. ω d. ω^2

35. If α is he cube root of unity, the for $n \in \mathbb{N}$, the value of $\alpha^{3n+1} + \alpha^{3n+5}$ is:
a. -1 b. 0 c. 1 d. 3

36. The complex number z is such that
$|z| = 1$, $z \neq -1$ and $\omega = \frac{z-1}{z+1}$, the real part of z is:
a. $\frac{1}{|z-1|^2}$
b. $-\frac{1}{|z-1|^2}$
c. $\frac{\sqrt{2}}{|z-1|^2}$
d. 0

37. $\lim_{x\to 1} x^{\frac{1}{x-1}}$ yields:
a. e b. 1 c. -1 d. 0

38. $\arg\left(\frac{1+i}{\sqrt{3}-i}\right)$ is:
a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{5\pi}{4}$ d. $\frac{\pi}{12}$

39. The vertices of a triangle are the points $P(1, 2, 1), Q(1, 0, 3), R(-1, 2, -1)$. Then the area of triangle PQR in square units is:
a. $4\sqrt{3}$ b. $2\sqrt{3}$ c. $4\sqrt{2}$ d. $2\sqrt{2}$

40. The value of $\int_0^2 (1 + [x])dx$, where $[x]$ denotes the greatest integer function is:
a. 3 b. 7 c. 2 d. 4

41. The value of a for which $f(x) = ae^{-2x}$ satisfies the differential equation $f''(x) - 3f'(x) - 4f(x) = 16e^{-2x}$, is:

a. $-\frac{8}{3}$ b. $\frac{8}{3}$ c. $\frac{3}{8}$ d. $\frac{8}{3}$

42. The root mean square value of the function $f(x) = \sqrt{x}$ for $x \in [0, 2]$ is:

a. $\frac{2\sqrt{2}}{3}$ c. $\sqrt{2}$

b. 1 d. $\frac{\sqrt{2}}{3}$

43. Given that for $n \geq 0, I_n = \int_0^1 x^n e^x dx$, then I_n yields:

a. $e - I_{n-1}$ c. $e - nI_{n-1}$

b. $e + I_{n-1}$ d. $e + nI_{n-1}$

44. The imaginary part of the complex number $z = 3e^{i\theta} + 2e^{-i\theta}$is:

a. $\cos\theta$ c. $\sin\theta$

b. $\cos 5\theta$ d. $\sin 5\theta$

45. There repeated decimal $0.432432432...$ expressed as a fraction is:

a. $\frac{432}{1000}$ b. $\frac{16}{37}$ c. $\frac{32}{43}$ d. $\frac{23}{34}$

Define a function g by $g(x) = |x|$ Answer questions 46 to 50.

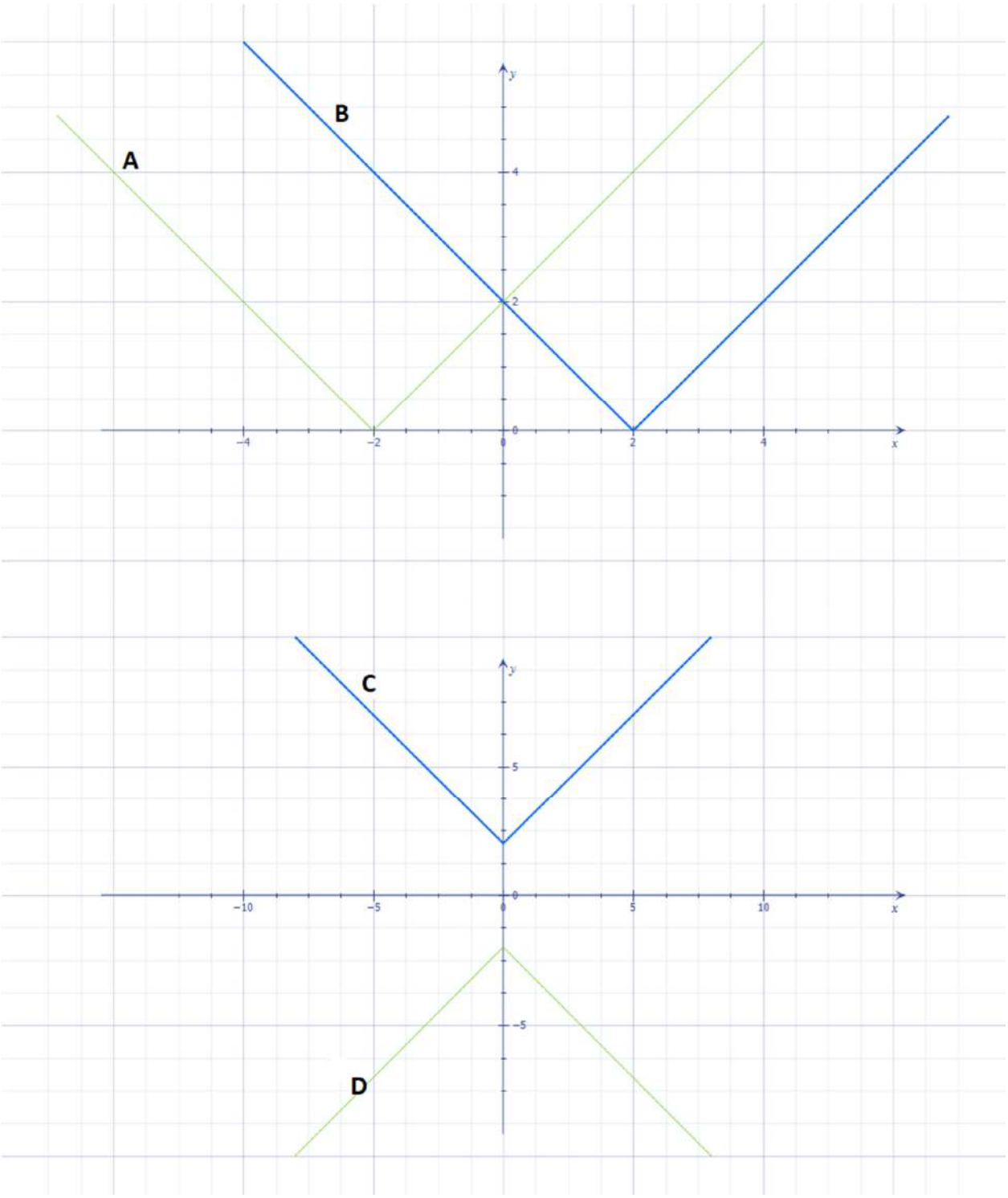
46. Which of the graphs represents the function $g(x - 2)$?

47. Which of the graphs represents the function $g(x + 2)$?

48. Which of the graphs represents the function $g(x) + 2$?

49. Which of the graphs represents the function $-g(x) - 2$?

50. Which of the graphs represents the function $g(2 - x)$?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